Name: _____

Circle True or False. (1 point for correct answer, 0 if incorrect)

1. True **FALSE** As long as we do not divide by 0, we have $\lim_{x\to c} f(x) = f(c)$.

Solution: The latter holds true if and only if f is continuous. But, there are examples of functions where this is not true.

2. True **FALSE** If we can find one horizontal line that hits the function at least once, then the function is surjective (onto).

Solution: In order for a function to be surjective, **all** horizontal lines must hit the function at least once.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (6 points) Find $\lim_{x\to\infty} \sqrt{x^2 - 4x + 1} - x$.

Solution: Multiplying by the conjugate gives

$$\lim_{x \to \infty} \frac{x^2 - 4x + 1 - x^2}{\sqrt{x^2 - 4x + 1} + x} = \lim_{x \to \infty} \frac{-4x + 1}{\sqrt{x^2 - 4x + 1} + x}.$$

Now dividing the top and bottom by x gives

$$= \lim_{x \to \infty} \frac{-4 + 1/x}{\sqrt{1 - 4/x + 1/x^2} + 1} = \frac{-4}{1 + 1} = -2.$$

(b) (4 points) Write the function that is x^3 shifted to the left by 3 then horizontally stretched by 2 and reflected across the *y*-axis. Then compressed vertically by a factor of 2 and shifted down by 2.

Solution: The horizontal transformation tells us that x is first divided by -2 then 3 is added. The vertical transformation tells us that the function is divided by 2 then 2 is subtracted. So the function is $(-x/2+3)^3/2 - 2$.